A Graphic Design Method for Matched Low-Noise Amplifiers

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Abstract —This paper presents a graphic design method for matched low-noise amplifiers where all necessary design information is presented in the load plane. It is possible to work exclusively in the load plane, as the input matching requirement makes the source admittance dependent on the load admittance. As a consequence of the bilinear transformation involved, all parameters may be presented by circles. Analytic equations giving the centers and the radii of the circles in the load plane are presented.

The method is particularly useful for feedback amplifiers. For a given feedback situation, a trade-off between different design parameters can be evaluated by inspection of the graph. The analytic equations make the calculations very fast, and the result of changes in the feedback element values can be viewed directly.

Two kinds of amplifier configurations are considered. First, a single-stage amplifier with an input match requirement is described. Secondly, a two-stage cascade amplifier is considered. The latter is required to have an output match and noise-optimized second stage, in addition to an input match. For the single stage case the noise figure, the power gain, the stability, and the input network are treated. In the cascade design, the total noise figure, the interstage network, and the available gain are treated as well. A design example for lossless feedback is presented.

I. Introduction

An ACTIVE device is normally mismatched at the input when it is noise optimized. As the input match is usually required, the amplifier has to be balanced or an isolator has to be used. These approaches have a loss and a size penalty. A third alternative is to use feedback to obtain the simultaneous match of noise and power.

The noise factor for the two-port in Fig. 1 may be written [1] as

$$F = F_{\min} + \frac{4r_n|\Gamma_S - \Gamma_{\text{opt}}|^2}{(1-|\Gamma_S|^2)|1+\Gamma_{\text{opt}}|^2}. \tag{1}$$

The noise factor is independent of the output reflection coefficient Γ_L.

The input reflection coefficient is given by

$$\Gamma_{\text{IN}} = S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1-S_{22}\Gamma_L} \tag{2}$$

and can be changed without affecting the noise figure. Input power match can therefore be achieved at minimum noise figure by choosing the appropriate Γ_L. The

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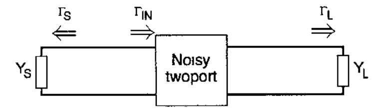

Fig. 1. Single stage amplifier

condition is

$$\Gamma_{\text{IN}}(\Gamma_L) = \Gamma^*_{\text{opt}}. \tag{3}$$

In many cases this approach is difficult to apply. The simultaneous matching condition may require a negative real part of the load admittance, or the gain may be too low to be of practical use. Furthermore, the required load admittance may cause the two-port to become unstable. It was noted by Rothe and Dahlke [2] that these problems can be avoided by applying feedback to the two-port. This was further expanded by Engberg [3]. Using computer optimization, Engberg made a graph from which, for a pair of lossless shunt and series feedback elements, a Γ_L could be found which fulfilled $\Gamma_{\text{IN}} = \Gamma^*_{\text{opt}}$. Though a pioneering work, the method gave little control of the design. For a given feedback situation it was not possible to make a design trade-off between different parameters.

This paper makes a trade-off possible by requiring only an input power match. A good view of the responses may be obtained by presenting constant parameter circles in the Smith chart [4]. However, as the design parameters are situated in both the load and source plane, it is hard to overview the design trade-offs. The new approach suggested in this paper is to move all design parameters to the same plane by using the input match requirement. The design trade-offs may then be directly evaluated by inspection of a single graph. Analytic equations for the single-stage design are presented. The single-stage design has the disadvantages of a low gain and output mismatch. A two-stage cascade, where the second stage is noise optimized and both the input and output are matched, has therefore been treated (Fig. 2). The interstage matching network of Fig. 2 is the output matching network of the first active device, as well as the input matching network of the second active device. Feedback is included in the first active device.

In the cascaded case, the major problem is that the interstage network has to be known in order to calculate the total noise figure and available gain. A lossless, recip-

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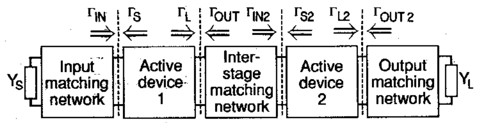

Fig. 2. Cascaded amplifier.

rocal interstage network has therefore been required to noise optimize the second stage. The interstage network is then underdetermined, and there is an infinite number of solutions.

A new circle in the load plane is derived which represents those load reflection coefficients for which the interstage requirements may be fulfilled. The first active two-port is in this case approximated to be unilateral.

It is now possible to derive the circle equations for the total noise figure and available gain. Once the Γ_L has been decided, the S parameters of the interstage matching network are calculated from a new set of equations.

This method is meant to be a higher level tool with which we can overview the possible designs and supply a network analysis program with data for simple networks to be realized. Optimization of complex networks with unknown global minima is in this way avoided.

The circle method is inherently a single-frequency method. This may be circumvented by finding the required S parameters of the matching networks for more than one frequency point, with the same feedback applied.

II. Single Stage

Let us first consider an example of how we can make a design trade-off between gain and noise by using existing methods. We start by finding a feedback situation where we have a passive Γ_L for the simultaneous match of input and noise figure. This could either be done by optimization or by trial and error. By inspecting the noise circles in the source plane a new Γ_S is picked. Using the input match requirement, Γ_L is calculated and we obtain the power gain from the circles in the load plane.

It is now obvious that by using this method we have no way of predicting the change in power gain for a given change in the noise figure. We would have to try different Γ_S to get a knowledge of the trade-offs that are possible for this particular feedback. This paper solves this problem by using the input match requirement to map either group of curves to the other plane. Fig. 3 illustrates how noise figure circles of the source plane can be mapped to the load plane and plotted, along with power gain circles. It is possible to do this on both planes, but only the load plane is of practical use. If the source plane is used, only part of this plane represents the passive load plane. This workable area is furthermore moving, rotating, and changing in size with applied feedback. This is illustrated in Fig. 4, where the entire passive load plane has been mapped to the source plane for three different series-feedback inductances.

The design approach is now different with the new method. Without feedback, we do not have a passive Γ_L

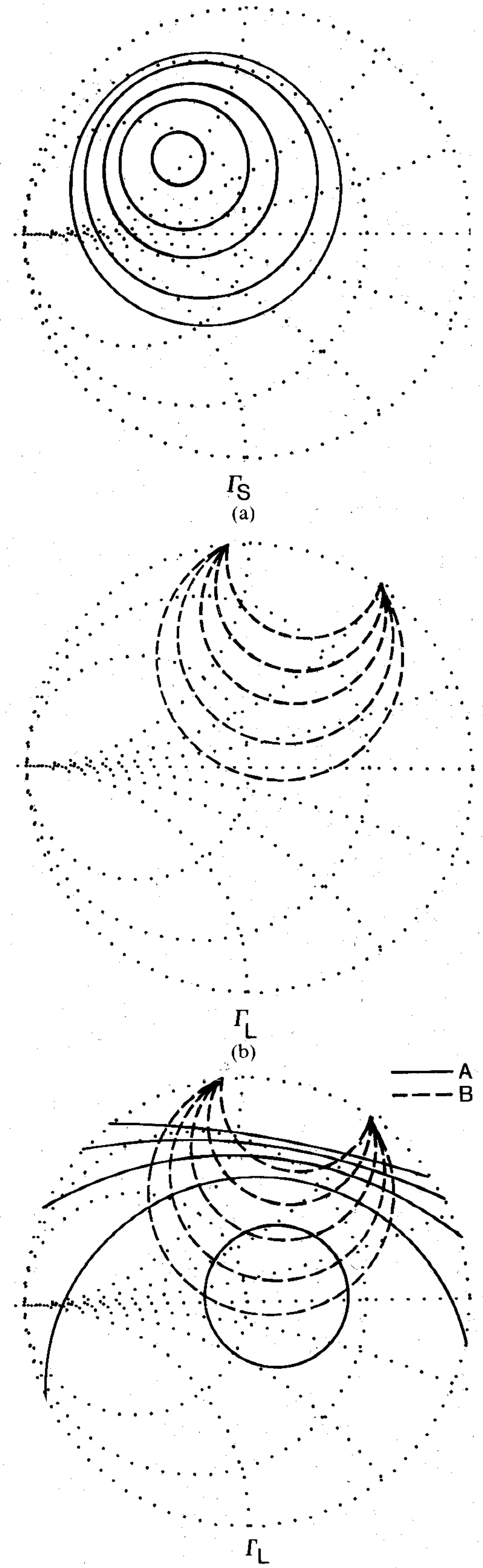

Fig. 3. Examples of noise figure circles and power gain circles for NEC FET NE72089, with a 0.5 nH series inductance feedback applied at 5 GHz. (a) Noise figure plotted in source plane admittance chart. Center circle 1.2 dB, and then a 0.25 dB increase for each new circle. (b) Power gain plotted in load plane admittance chart. Center circle 10.5 dB, and then a 1 dB decrease for each new circle. (c) Noise figure A and power gain B plotted together in load plane admittance chart.

which will simultaneously match the amplifier; i.e., the noise circle "center" is outside of the passive load plane. However, part of the noise circles for higher noise figures are within the passive plane. If we now apply feedback to the amplifier we can use their movements to guide the "center" into the passive plane. Since both the power gain circles and noise circles are in the same plane, we can directly evaluate different trade-offs by inspecting the graph. Stability can be evaluated at the same time by plotting the two stability circles in the load plane as well. The unit circle and the real axis of the source plane represent simple input circuits.

Since this graphical approach is intended to be implemented on a computer, it is essential to be able to plot the curve groups rapidly. The centers and radii of the curves have therefore been derived analytically. To make the notation compact, the results have been expressed in a special notation found in the Appendix.

The circle for the constant noise figure F_i is, using (1), derived to be,

$$\begin{cases} A = |X|^2 - N_i R & (4) \\ B = -|Y|^2 + N_i V & (5) \\ C = X^* Y + N_i T & (6) \end{cases}$$

$$N_i = \frac{(F_i - F_{\min})}{N}. \quad (7)$$

The power gain G_p is already defined in the load plane:

$$G_P = \frac{1}{1-|\Gamma_{IN}|^2}|S_{21}|^2 \frac{1-|\Gamma_L|^2}{|1-S_{22}\Gamma_L|^2} \quad (8)$$

$$\begin{cases} A = -R + \dfrac{|S_{21}|^2}{G_P} & (9) \\ B = V + \dfrac{|S_{21}|^2}{G_P} & (10) \\ C = T & (11) \end{cases}$$

The input stability circle is moved to the load plane, while the output stability circle is already defined there:

$$\text{Input} \begin{cases} A = R^2 - |T|^2 & (12) \\ B = V^2 - T^2 & (13) \\ C = T(R - V) & (14) \end{cases}$$

$$\text{Output} \begin{cases} A = R & (15) \\ B = -V & (16) \\ C = -T. & (17) \end{cases}$$

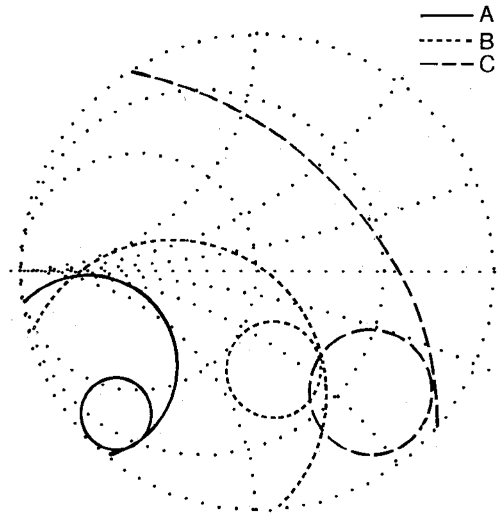

Fig. 4. Load plane $|\Gamma| = 1$ circle and unit conductance circle mapped to source plane admittance chart for NEC FET NE72089 with: A—no feedback; B—0.7 nH series feedback; and C—1.8 nH series feedback.

The unit circle and real axis of the source plane are transformed to the load plane, yielding:

$$\text{Unit circle} \begin{cases} A = \text{Re}[S_{11}] - |S_{11}|^2 & (18) \\ B = |\Delta|^2 - \text{Re}[S_{22}\Delta^*] & (19) \\ C = \dfrac{\Delta + S_{11}^* S_{22} + 2S_{11}^*\Delta}{2} & (20) \end{cases}$$

$$\text{Real axis} \begin{cases} A = \text{Im}[S_{11}^*] & (21) \\ B = \text{Im}[S_{22}^*\Delta] & (22) \\ C = \dfrac{j}{2}(S_{11}^* S_{22} - \Delta). & (23) \end{cases}$$

III. Cascaded Case

The single-stage design has several drawbacks. The gain is low, thereby making the next stage significant in noise calculations. The output is not matched, thereby making it hard to cascade several equal stages. One way of avoiding these drawbacks is to include a second stage in the design. Only the first active device need to have the feedback applied.

To be able to derive circles in the load plane for the total noise figure and total gain, we need to have some knowledge of the interstage network. We desire a reciprocal, lossless interstage network which optimizes the second-stage noise. The whole cascade should be matched at the input and at the output.

The interstage requirements are then:

Noise Match:

$$\Gamma'_{\text{opt}} = \frac{S_{22}^I - \Delta^I \Gamma_{\text{OUT}}}{1 - S_{11}^I} \quad (24)$$

Reciprocity:

$$S_{21}^I = S_{12}^I \quad (25)$$

Losslessness [6]:

$$\begin{cases} |S_{11}^I|^2 + |S_{21}^I|^2 = 1 & (26) \\ |S_{12}^I|^2 + |S_{22}^I|^2 = 1 & (27) \\ S_{11}^I S_{12}^{I*} + S_{21}^I S_{22}^{I*} = 0. & (28) \end{cases}$$

The equation system is underdetermined. There exists therefore an infinite number of interstage networks which fulfil the requirements. The Γ_L of these networks will give a circle in the load plane. The center and radius of this circle has been derived under the assumption that the first active device is unilateral:

$$\begin{cases} A = \left|\Gamma_{IN2} - \frac{1}{\Gamma_{opt2}}\right|^2 - \left|\frac{1}{\Gamma_{OUT}}\left(1 - \frac{\Gamma_{IN2}}{\Gamma_{opt2}^*}\right)\right|^2 & (29) \\ B = \left|1 - \frac{\Gamma_{IN2}}{\Gamma_{opt2}^*}\right|^2 - \left|\frac{1}{\Gamma_{OUT}}\left(\Gamma_{IN2} - \frac{1}{\Gamma_{opt2}}\right)\right|^2 & (30) \\ C = \frac{1}{\Gamma_{OUT}^*}\left(1 - |\Gamma_{IN2}|^2\right)\left(1 - \frac{1}{|\Gamma_{opt2}|^2}\right). & (31) \end{cases}$$

Once a certain Γ_L has been chosen from this circle, the interstage s-parameters may be calculated, using:

$$\begin{cases} S_{22} = \frac{ue^{j2p} - \Gamma_L + v}{t} & (32) \\ S_{11} = -S_{22}^* e^{j2p} & (33) \\ S_{12} = \sqrt{\left(1 - |S_{22}|^2\right)e^{j2p}} & (34) \\ S_{21} = S_{12} & (35) \end{cases}$$

where

$$e^{j2p} = \frac{vxst^* - tu^* - v|t|^2}{s^*t - t^*vxu - x|t|^2} \quad (36)$$

$$s = v - \Gamma_L \quad (37)$$

$$t = vx - \Gamma_L \Gamma_{IN2} \quad (38)$$

$$u = \Gamma_{IN2} - x \quad (39)$$

$$v = \frac{1}{\Gamma_{OUT}} \quad (40)$$

$$x = \frac{1}{\Gamma_{opt2}}. \quad (41)$$

These expressions are exact when the source input admittance is equal to the normalizing admittance. Even for source admittances far from the normalizing admittance, the approximation has been found to have small influence in the case of FET amplifiers. The approximation error is easily calculated once a Γ_L has been decided upon.

As the second stage now is assured the minimum noise reflection coefficient at its input, we are able to consider the gain and noise of the cascaded amplifier. The available gain of the second stage is equal to its associated gain. All the matching networks are assumed to be lossless and reciprocal. Thus their available gains equal unity. The total available gain of the cascade is then the product of the individual available gains. The changes in the total available gain will therefore exclusively be governed by the available gain of the first active device:

$$G_a = \frac{|S_{21}|^2\left(1 - |\Gamma_S|^2\right)}{|1 - S_{11}\Gamma_S|^2 - |S_{22} - \Delta\Gamma_S|^2}. \quad (42)$$

The input match requirement is used to derive the circle parameters,

$$\begin{cases} A = R + \frac{G_a}{|S_{21}|^2}\left(|T|^2 - R^2\right) & (43) \\ B = -V + \frac{G_a}{|S_{21}|^2}\left(|T|^2 - V^2\right) & (44) \\ C = -T + \frac{G_a}{|S_{21}|^2}T(R - V). & (45) \end{cases}$$

Notations are as in the Appendix.

The second stage is noise optimized; thereby the total noise figure, using Friis's formula [5], can be expressed as

$$F_{tot} = F(\Gamma_S) + \frac{F_{min2} - 1}{G_a(\Gamma_S)}. \quad (46)$$

Using the noise figure of (1) and the available gain of (42), the load plane parameters are derived as follows:

$$\begin{cases} A = -MR + N|X|^2 + O\left(R^2 - |T|^2\right) & (47) \\ B = MV - N|Y|^2 + O\left(V^2 - |T|^2\right) & (48) \\ C = MT + NYX^* + OT(V - R) & (49) \end{cases}$$

where $M = F_{tot} - F_{min}$.

The stability of the second stage is ensured by the optimum noise input admittance and output match.

TABLE I

G_a	12.59 dB	inter S_{11}	$0.69\angle 56.21°$
F_{tot}	0.65 dB	inter S_{12}	$0.72\angle -28.18°$
Γ_L	$0.86\angle 122.54°$	inter S_{22}	$0.69\angle 67.43°$
Y_S	$0.22 - j0.69$		

IV. Design Example

In this cascaded amplifier design example a NEC 75083 with a minimum noise figure of 0.6 dB and an associated gain of 10.5 dB at 5 GHz is used. Inductive series feedback is applied to bring the center of the total noise figure circles into the load plane. By adjustment of this inductance, the minimum total noise figure is moved on top of the interstage circle.

Each active device has to be ascertained for stable input and output admittances for not only the design frequency. The first active device is more stable because of the feedback and is usually no problem. The most critical admittance is the output admittance of the interstage network. One way of reducing this problem is not to choose the Γ_{opt}, but rather a source reflection coefficient for the second active device that is closer to the source plane center. Noise performance is then sacrificed to simplify the interstage design. The degradation of the noise factor by 0.6 dB in the second stage results in the total noise factor degradation by 0.05 dB. The most important data obtained from the computer program are given in Table I.

The total available gain is obtained as the sum of the available gain G_a and the associated gain. Observe that the noise figure is not invariable to lossless feedback, as opposed to the noise measure. Therefore a minimum noise decrease in the first stage is observed.

Using the commercial program Touchstone, an interstage network was optimized to yield the calculated S parameters. The active devices and feedback inductor were added and an input and an output matching network were optimized for input and output power match (Fig. 5). Touchstone calculations give the following response of the complete amplifier at 5 GHz:

$$\Gamma_L = 0.83\angle 118.5°$$
$$F_{tot} = 0.78 \text{ dB}$$
$$|S_{11}| = -28.5 \text{ dB}$$
$$|S_{22}| = -23.5 \text{ dB}$$
$$|S_{21}| = 22.5 \text{ dB}.$$

Observe that no noise optimization has to be made. The deviations are a result of the approximation made.

V. Computer Program

The computer program was originally written in Fortran for a HP9000/550 with a 2627A color terminal. To be able to run it on a more common system, the program has been moved to an IBM PC environment using the EGA and a mouse. This program is available from the author by sending a formatted 3-1/2 in. disk.

VI. Conclusion

A graphic design method for matched low-noise amplifiers has been presented. Single-stage and two-stage cascade designs have been treated. The method presents information on the complete amplifier response for each feedback situation. This is done by plotting constant parameter circles in the load plane of the first stage.

Appendix

The centers and radii of the circles in the load plane are derived by transforming the constant parameter equations into the form:

$$|\Gamma_L|^2 B - 2\,\mathrm{Re}[\Gamma_L C] = A \tag{A1}$$

where A and B are real, and C is complex.

This is the well-known circle equation from which we identify:

$$\text{Center} \quad \Gamma_{LC} = \frac{C^*}{B} \tag{A2}$$

$$\text{Radius} \quad R_L = \frac{\pm\sqrt{BA + |C|^2}}{B}. \tag{A3}$$

Once the equation has been transformed into the form of (A1), the radius and center are directly obtained using

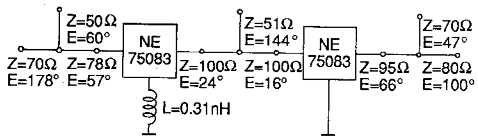

Fig. 5. Circuit of cascaded amplifier design.

(A2) and (A3). As certain combinations of variables reappear in many of the circle derivations, it has been found convenient to use the following notation:

$$R = 1 - |S_{11}|^2 \tag{A4}$$
$$T = S_{22} - S_{11}^*\Delta \tag{A5}$$
$$X = \Gamma_{opt}^* - S_{11} \tag{A6}$$
$$Y = \Delta - S_{22}\Gamma_{opt}^* \tag{A7}$$
$$V = |S_{22}|^2 - |\Delta|^2 \tag{A8}$$
$$N = \frac{4R_n}{|1+\Gamma_{opt}|^2} \tag{A9}$$
$$O = \frac{F'_{min} - 1}{|S_{21}|^2} \tag{A10}$$

where $\Delta = S_{11}S_{22} - S_{12}S_{21}$ and F'_{min} is the minimum noise figure of the second active device.

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